CS 798: Homework Assignment 2 (Probability)

Assigned: September 30, 2009

1.0 Sample space

In the IEEE 802.11 protocol, the congestion window (CW) parameter is used as follows: initially, a terminal waits for a random time period (called *backoff*) chosen in the range $[1, 2^{CW}]$ before sending a packet. If an acknowledgement for the packet is not received in time, then CW is doubled, and the process is repeated, until CW reaches the value CWMAX. The intial value of CW is CWMIN. What is the sample space for (a) the value of CW? (b) the value of the the backoff?

2.0 Interpretations of probability

Consider the statement: given the conditions right now, the probability of a snowstorm tomorrow morning is 25%. How would you interpret this statement from the perspective of an objective, frequentist, and subjective interpretation of probability (assuming these are possible)?

3.0 Conditional probability

Consider a device that samples packets on a link. (a) Suppose that measurements show that 20% of packets are UDP, and that 10% of all packets are UDP packets with a packet size of 100 bytes.What is the conditional probability that a UDP packet has size 100 bytes? (b) Suppose 50% of packets were UDP, and 50% of UDP packets were 100 bytes long. What fraction of all packets are 100 byte UDP packets?

4.0 Conditional probability again

Continuing with Ex. 3: How does the knowledge of the protocol type change the sample space of possible packet lengths? In other words, what is the sample space before and after you know the protocol type of a packet?

5.0 Bayes' rule

For Exercise 3(a), what additional information do you need to compute P(UDP|100)? Setting that value to x, express P(UDP|100) in terms of x.

6.0 Cumulative distribution function

(a) Suppose discrete random variable D take values {1, 2, 3, ...,i,...} with probability $1/2^i$. What is its CDF?

(b) Suppose continuous random variable C is uniform in the range $[x_1, x_2]$. Whats is its CDF?

7.0 Expectations

Compute the expectations of the D and C in Exercise 6.

8.0 Variance

Prove that $V[aX] = a^2V[X]$.

9.0 Bernoulli distribution

A hotel has 20 guest rooms. Assuming outgoing calls are independent and that a guest room makes 10 minutes worth of outgoing calls during the busiest hour of the day, what is the probability that 5 calls are simultaneously active during the busiest hour? What is the probability of 15 simultaneous calls?

10.0 Geometric distribution

Consider a link that has a packet loss rate of 10%. Suppose that every packet transmission has to be acknowledged. Compute the expected number of data transmissions for a successful packet+ack transfer.

11.0 Poisson distribution

Consider a binomially distributed random variable X with parameters n=10, p=0.1. (a) Compute the value of $P(X=8)$ using both the binomial distribution and the Poisson approximation. (b) Repeat for n=100, p=0.1

12.0 Gaussian distribution

Prove that if X is Gaussian with parameters (μ, σ^2), then the random variable $Y=aX + b$, where a and b are constants, is also Gaussian, with parameters$(a\mu + b, (a\sigma)^2)$. Hint: Consider the cumulative distribution for Y.

13.0 Exponential distribution

Suppose that customers arrive to a bank with an exponentially distributed inter-arrival time with mean 5 minutes. A customer walks into the bank at 3pm. What is the probability that the next customer arrives no sooner than 3:15?

14.0 Exponential distribution

It is late August and you are watching the Perseid meteor shower. You are told that that the time between meteors is exponentially distributed with a mean of 200 seconds. At 10:05 pm, you see a meteor, after which you head to the kitchen for a bowl of icecream, returning outside at 10:08pm. How long do you expect to wait to see the next meteor?

15.0 Power law

Consider a power-law distribution with $x_{min} = 1$ and $\alpha = 2$ and an exponential distribution with $\lambda = 2$. Fill in the following table:

x	$f_{power\_law}(x)$	$f_{exponential}(x)$
1		
5		
10		
50		
100		

16.0 Markov's inequality

Consider a random variable X that exponentially distributed with parameter $\lambda = 2$. What is the probability that $X > 10$ using (a) the exponential distribution (b) Markov's inequality.

17.0 Joint probability distribution

Consider the following probability mass function defined jointly over the random variables, *X, Y,* and *Z*: *P(000) = 0.05; P(001) = 0.05; P(010) = 0.1; P(011)=0.3;P(100) = 0.05; P(101) = 0.05; P(110) = 0.1; P(111)=0.3.* (a) Write down p_X, p_Y, p_z, p_{XY}, p_{XZ}, p_{YZ}. (b) Are *X* and *Y*, *X* and *Z*, or *Y* and *Z* independent? What is the probability that *X=0* given that *Z=1*?