CS 798: Homework Assignment 4 (Game Theory)

Assigned: October 28, 2009

1.0 Preferences

Suppose that you equally like a banana and a lottery that gives you an apple 30% of the time and a carrot 70% of the time. Also, you equally like a peach and a lottery that gives you an apple 10% of the time and a carrot 90% of the time. (a) What can you say about your relative preferences for bananas and peaches? If you had a lottery whose payoffs were bananas and carrots, what probability of winning a banana or a carrot would be equally preferred to a peach?

2.0 Utility functions

Your cable company gives you 10GB of free data transfer a month, and charges $5/GB thereafter. Suppose that your utility from transferring x GB of data is $100(1-e^{-0.25x})$ and that your disutility from paying $1 is 1. How much data should you transfer in a month to maximize your utility?

3.0 Pure and mixed strategies

Consider the game of tic-tac-toe. What are the possible actions for the first move of the first player (ignore symmetries)? What would constitute a pure strategy? What would constitute a mixed strategy? Would you ever play a mixed strategy for this game? Why or why not?

4.0 Zero-sum game

If the payoffs (a, $-a$) of every outcome of a zero sum game were changed so that the new payoffs were (a+5, -5a), the game would no longer be zero sum. But, would the structure of the game change?

5.0 Representation

Represent the Pricing game of Example 7 in Normal form.

6.0 Representation

Prove that normal and extensive form are equivalent if information sets are permitted.

7.0 Best response

What is the best response for the customer in the Pricing game (Example 7)?

8.0 Dominant strategy

Suppose that you are not well prepared for a final, and you think you might fail it. If you miss the exam, you will certainly fail it. What is your dominant strategy: attend or miss? Why?

9.0 Bayesian game

Does the Bayesian game in Example 11 have a dominant strategy for the Row player? If so, what is it?

10.0 Repeated game

Suppose that both players in Prisoner's Dilemma (Example 15) play their dominant strategy in an infinitely repeated game with a discount factor of 0.6. What is their payoff for the repeated game?

11.0 Dominant strategy equilibrium

Interpret the meaning of the dominant strategy equilibrium of Example 14. Look up how the 802.11e EDCA protocol solves this problem.

12.0 Iterated deletion

Show an example of a pure strategy that is dominated by a mixture of other pure strategies, although none of the strategies in the mixture dominate the pure strategy.

13.0 Maximin

What are the maximin equilibria in Examples 10 and 14?

14.0 Maximin in a zero-sum game

Show that in Example 18 if Row uses any value of p other than 0.5, then it may get a payoff lower than 2.5 if Column plays either pure or mixed strategies.

15.0 Nash equilibrium

Referring to Example 19, assume that if Column plays a mixed strategy with probability qH + (1-q)T instead of its Nash equilibrium strategy. What is Row's mixed strategy best response?

16.0 Correlated equilibrium

Does the WiFi game of Example 6 have a correlated equilibrium? If so, describe it.

17.0 Price discrimination

Outline the design of a price-discrimination mechanism with n player types (whose valuations are known).

18.0 VCG mechanism

The CIO of a company wants to decide how much capacity to buy from its ISP. The cost of capacity is \$20/Mbps/month. There are three departments in the company, who value capacity as follows: department 1 (D1) values capacity xMbps/month at $\$20(*1-e^{-0.5x})$, D2 values it at $\$40(*1-e^{-0.5x})$, D3 values it at $\$80(*1-e^{-0.5x})$. (a) Assuming the disutility of ISP payment is linear in the amount of payment, what is the overall function that the CIO should maximize? (b) What is type of each department? (c) What is the optimal social choice? (d) What are the Clarke Pivot payments for each department? (e) Is this budget balanced?