CS 798: Homework Assignment 1 (Optimization)

Assigned: September 16, 2009

1.0 Modelling

You have been hired as the head of CHYM FM's balloon operations. Too much money is being spent for each flight! Your job is to make flight profitable again (the number of flights is not negotiable).

For each flight, you can control where you take off from (there is a finite set of take-off locations) and the duration of the flight, as long as the flight lasts at least 15 minutes. The cost of a flight depends on its duration (to pay for natural gas, the pilot's wages, and for the chase vehicle), where the balloon takes off from, and how far the landing site is from a road (the further away it is from a road, the more it has to be dragged over a farmer's field). Moreover, you can have up to 9 passengers (in addition to at least one pilot), and charge them what you wish. Of course, the number of passengers decreases (say linearly) with the cost of the ticket.

What are the fixed parameters? What are the input and output parameters? What are the control variables? Come up with plausible transfer and objective functions. How would you empirically estimate the transfer function?

2.0 Optimizing a function of two variables

Consider the following system:

$$O = 10x_1 - 3x_2, \text{ where}$$

$$2x_1 - x_2 = 1 \quad and$$

$$x_1 \geq 0 \; x_2 \geq 0$$

Geometrically find the optimal value of *O*.

3.0 Optimizing a function of three variables

Geometrically find the optimal value of *O* where

$$O = 5x_1 + 2x_2, - x_3 \text{ and} \qquad (EQ\ 1)$$

$$x_1 + x_2 + x_3 = 1 \text{ and} \qquad \text{(EQ 2)}$$

$$x_1 \geq 0 \; x_2 \geq 0 \;\; 3 \geq 0 \qquad (EQ\ 3)$$

4.0 Representing a linear program

Make and state the appropriate assumptions to model the system in Problem 1 in standard form.

5.0 Network flow

Model the network flow problem where the warehouses have finite bounded capacity as a linear program.

6.0 Integer linear programming

Generalize Example 6 to the case where *n* users can schedule jobs on one of *k* machines, such that each user incurs a specific cost and gains a specific benefit on each machine at each of *m* time periods. Write out the ILP for this problem.

7.0 Weighted bipartite matching

Suppose you have *K* balls that need to placed in *M* urns such that the payoff from placing the *k*th ball in the *m*th urn is p_{km}, and no more than 2 balls can be placed in each urn. Model this as a weighted bipartite matching problem.

8.0 Dynamic programming

You are given a long string *L* of symbols from a finite alphabet. Your goal is to find matches of a substring of *L* with a shorter string *S* from the same alphabet. The catch is that match need not be exact: each non-matching symbol gets a penalty of 1. So, you can have a trivial 'match' with a penalty of $|S|$. What you want is to output all matching substrings of *L* along with the corresponding penalty. Use dynamic programming to solve this problem.

9.0 Lagrangian optimization

Use Lagrangian optimization to find the optimal value of $z=x^3 + 2y$ subject to the condition that $x^2+y^2 = 1$ (i.e. the points (x,y) lie on the unit circle.

10.0 Hill climbing

Suppose you know that the objective function you are trying to maximize has no more than *K* local optima. Outline an algorithm that is guaranteed to find the global optimum using hill climbing.